On the Number Density of Sunyaev–Zel'dovich Clusters of Galaxies.

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ABSTRACT

If the mean properties of clusters of galaxies are well described by the entropy-driven model, the distortion induced by the cluster population on the blackbody spectrum of the Cosmic Microwave Background radiation is proportional to the total amount of intracluster gas while temperature anisotropies are dominated by the contribution of $10^{14} M_\odot$ clusters. This result depends marginally on cluster parameters and it can be used to estimate the number density of clusters with enough hot gas to produce a detectable Sunyaev-Zel'dovich effect. Comparing different cosmological models, the relation depends mainly on the density parameter Ω_m. If the number density of clusters could be estimated by a different method, then this dependence could be used to constrain Ω_m.

Subject headings: Cosmic Microwave Background. Cosmology: theory. Galaxies: clusters: general.

1. Introduction.

Clusters of galaxies can be detected in the X-ray band due to the emission of the intracluster (IC) gas. This gas is hot enough to change the brightness of the Cosmic Microwave Background (CMB) photons through inverse Compton scattering (Zel'dovich & Sunyaev 1969). The Sunyaev-Zel'dovich (SZ) effect caused by individual clusters has been measured for tens of clusters (see Birkinshaw 1999 for a review). Eventually, the PLANCK satellite will produce an all-sky catalogue likely to contain thousands of SZ sources (da Silva et al. 1999). Together with the effect of single sources, the overall cluster population induces distortions and temperature anisotropies on the CMB. The Sunyaev-Zel'dovich effect has a well known frequency dependence that helps to distinguish it from other foregrounds and methods have already been proposed to separate and measure this contribution (Hobson et al. 1998).

Extensive theoretical work has been devoted to analyze the effect of clusters on the CMB radiation (Cole & Kaiser 1988, Bartlett & Silk 1994, Colafrancesco et al. 1994, Atrio-Barandela & Mücket 1999 -hereafter paper I-, Komatsu & Kitayama, 1999). Barbosa et al. (1996) noticed that the value of the mean Comptonization parameter, that measures the amplitude of the blackbody distortion, depends on which are the less massive clusters that produce a significant effect. On the other hand, Cole & Kaiser (1988) remarked that temperature anisotropies were dominated by massive clusters at moderate redshifts. Since clusters of a given mass contribute differently to temperature anisotropies and distortions, in this letter we show how the number density of clusters with enough hot gas to produce a detectable SZ effect can be estimated. This number is marginally dependent on the cluster model and it varies by at most a factor of four in different cosmologies.

2. Scaling relations of clusters.

Following paper I, we assume that clusters are spherical in shape, with typical size the virial radius r_v, virial mass M and have virialized at redshift z. We also assume that the IC gas is isothermal, distributed smoothly and with a radial distribution well fitted by the spherical isothermal β model $n_e(r) = n_c[1 + (r/r_c)^2]^{-3\beta/2}$ (Jones & Forman 1984), where n_c, r_c are the central electron density and the core radius of the cluster, respectively. For conve-

nience, we shall assume that $\beta = 2/3$. Similar results to those presented here are obtained for any value of β within the observed range ($0.5 \leq \beta \leq 0.7$, see Markevitch et al. 1997). Further, we assume that the dynamical evolution of the IC gas is well described by the entropy-driven model by Bower (1997) since this model seems to provide an adequate description of clusters with redshifts $z \leq 0.4$ (Mushotzky & Scharf 1997).

To compute the spectral distortion and temperature anisotropies induced by hot IC gas we need to translate the properties of a sample of clusters at low redshifts into their equivalents at earlier epochs. For the spherical collapse model, the virial radius scales with mass and redshift as $r_v = r_{vo}(M/10^{15}M_\odot)^{1/3}(1+z)^{-1}$. If the electron temperature is proportional to the velocity dispersion of the dark matter then $T = T_g(M/10^{15}M_\odot)^{2/3}(1+z)$. In the entropy-driven model, the electron central density scales as $n_c = n_{co}(T/T_g)^{3/2}(1+z)^{-3\epsilon/2}$. Finally, the core radius scales as $r_c = r_{co}(M/10^{15}M_\odot)^{-1/6}(1+z)^{(-1+3\epsilon)/4}$. In this scaling relations, $r_{co}, r_{vo}, T_g, n_{co}$ are the current average core and virial radius, the central gas temperature and electron density of a $10^{15}M_\odot$ cluster, respectively, while ϵ parametrizes the rate of core entropy evolution. Mushotzky and Scharf (1997) found $\epsilon = 0 \pm 0.9$.

3. Distortions and Temperature anisotropies.

The effect of the IC gas is both to distort the blackbody spectrum and to induce temperature anisotropies on the CMB. A measure of such distortion is the mean Comptonization parameter defined as

$$\bar{y} = g(hf/K_BT_o) \int \frac{dn}{dM}dM\frac{dV}{dz}dz\kappa y_o\bar{\phi}. \quad (1)$$

In this expression $g(x) = x\coth(x/2) - 4$ gives the dependence of the SZ effect with frequency f; T_o is the CMB mean temperature, dn/dM is the cluster number density per unit of mass, $y_o = [k_B\sigma_T/m_ec^2]r_cT_gn_c$ with physical constants having their usual meaning and $\kappa = (r_v/2d_A)^2$ gives the probability that a particular line of sight crosses a cluster, with d_A the angular distance to the cluster. If we introduce $p = r_v/r_c$ then $\bar{\phi} = 4p^{-2}(p - \tan^{-1}p)$ is the averaged line of sight through a cluster.

The effect of all clusters on an angular scale l can be obtained by adding in quadrature the contribution of each single cluster. If we assume that clusters are

Poisson distributed on the sky the radiation power spectrum is (Cole & Kaiser 1988)

$$P(l) = \int \frac{dn}{dM} dM \frac{dV}{dz} dz (g(x)y_o)^2 |\tilde{\phi}(l)|^2, \quad (2)$$

where $\tilde{\phi}(l)$ is the Fourier transform of the angular profile of the IC gas. This expression represents the contribution of the foreground cluster population to the power spectrum of CMB temperature anisotropies. The variance of the temperature field is given by

$$< (\frac{\Delta T}{T_o})^2 > = \frac{1}{2\pi} \int l dl W(l) P(l) \quad (3)$$

In this equation, $W(l)$ represents the window function of the experiment. This temperature anisotropy, together with other secondary contributions, adds in quadrature with the "intrinsic" (cosmological) signal to give the pattern of temperature anisotropies on the sky (see paper I for details).

To compute the number density of clusters as a function of mass and redshift we use the Press-Schechter approximation, known to describe accurately the abundance of clusters in numerical simulations at different redshifts

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} (\frac{\rho}{M}) \frac{\nu}{\sigma} \frac{d\ln\sigma}{dM} e^{-1/2(\nu/\sigma)^2}. \quad (4)$$

In this expression, $\nu = (\delta_v b/\sigma_8)$ is the peak-height threshold, σ is the rms mass fluctuation within a top-hat filter measured in units of σ_8, the rms mass fluctuation on a sphere of $8h^{-1}$Mpc today, δ_v is the threshold overdensity of the spherical collapse model and b is the bias factor.

4. The cluster number density.

We integrate numerically eqs. (1) and (3) for three cosmological models with the same baryon fraction $\Omega_B = 0.05$: two flat models, (1) standard CDM, with $\Omega_{cdm} = 0.95$, (2) ΛCDM with $\Omega_{cdm} = 0.25$ and $\Omega_\Lambda = 0.7$ as indicated by the recent measurements of high redshift supernovae (Perlmutter et al. 1999) and (3) an open model, OCDM, with $\Omega_{cdm} = 0.25$ and no cosmological constant, consistent with cluster dynamics and evolution (Bahcall 1999). As average parameters, for a 10^{15}M$_\odot$ cluster we took: $r_{vo} = 1.3h^{-1}$Mpc, $T_g = 10^8$K, $r_{co} = 0.13h^{-1}$Mpc and $n_{co} = 1.19 \cdot 10^{-3} h^{1/2}$cm^{-3}. We considered different gas evolutionary histories, by taking $\epsilon = -1, 0, 1$. The matter power spectrum was normalized to $\sigma(8h^{-1}\text{Mpc}) = 0.7$ (Einasto et al. 1999b) and we considered the following bias factors: $b = 1.05, 1.3$. The first corresponds to the bias of QDOT galaxies and the second is an average value obtained from several catalogues (Einasto et al 1999a). Also, we considered $\delta_v = 1.5, 1.7$ (Tozzi & Governato, 1998, Governato et al. 1999). The simplicity of the Press-Schechter approach lies on the fact that the number density depends only on the ratio $\nu = \delta_v b/\sigma_8$. With the values quoted above, ν varies in the range 2 to 3 and we shall quote our results for both limits.

To compute the effect of the cluster population on the CMB radiation it is necessary to determine first if groups of galaxies of about 10^{13}M$_\odot$ have enough hot gas to produce a significant contribution. The scaling relations of Sec. [2] have been found to be accurate for clusters above 10^{14}M$_\odot$. However, they could very well be extended below this mass scale. In this respect, it is necessary to specify the integration range of eqs. (1) and (2). We tried three different lower mass limits: $M_{min} = 0.1, 0.5$ and 1 (in units of 10^{14}M$_\odot$). The main conclusion was that when changing the mass limit one order of magnitude $\bar{y}$ could vary a factor 3 to 10, depending on the model, but the temperature anisotropy $\Delta T/T_o$ varied by less than 30%. In this calculations and in the rest of the paper, temperature anisotropies were computed using the window function of the SuZie experiment (Church et al. 1997). For the models considered above, the anisotropies in the Rayleigh-Jeans regime ranged from 14μK for sCDM with $\nu = 2$ to 1μK for ΛCDM with $\nu = 3$.

Integration of eq (2) shows that $l^2 P(l)$ reaches a maximum around $l = 1000 - 3000$. At those scales Komatsu & Kitayama (1999) have concluded that the increase in temperature anisotropy induced by cluster correlations is negligible. To illustrate the different behavior of temperature anisotropies and distortions with the lower mass integration limit we shall consider the behavior of the radiation power spectrum at $l = 1000$. This scale is close to the maximum of the SZ effect and is a good compromise between the smallest angular scale that will be resolved by PLANCK and the maximum of the SuZie window function. In Fig. 1 we show $d\bar{y}/dM, dP(l = 1000)/dM$ for different cosmological models. Notice that the integrand of $\bar{y}$ increases at the low mass end, but that of the radiation power spectrum reaches a maximum around 10^{14}M$_\odot$. Its exact location is weakly dependent on model parameters. A qualitative explanation of Fig. 1 can be

given: since $\sigma \sim 1$ on a mass scale of $\approx 5 \times 10^{14} \mathrm{M}_\odot$, in the upper mass limit $d\bar{y}/dM$ and dP/dM are damped by the exponential factor in the Press-Schechter formula; in the low mass limit, the exponential factor is close to unity and $d\bar{y}/dM \sim M^{(1+n)/6}$ but $dP/dM \sim M^{n/6+3/2}$ where n is the slope of the matter power spectrum at $10^{13} \mathrm{M}_\odot$, roughly $n \simeq -2$ (Einasto et al. 1999a).

To conclude, the gas in groups contributes significantly to Comptonization but little to temperature anisotropies. This fact can be used to estimate the number density of clusters that produce a significant SZ effect. For this purpose, in paper I we introduced the parameter $\eta = \frac{\Delta T/T_o}{\bar{y}}$. It represents a relation similar to the approximate solution of the full kinetic equation for the change of photon distribution due to inverse Compton scattering for a single cluster: $\Delta T/T_o = g(hf/kT_o)y_o$. As an example, in Fig. 2a we plot η with respect to lower mass integration limit for three cosmological models with $\epsilon = 0$: solid line (sCDM), dot-dashed line (OCDM) and dashed line (ΛCDM). Thick lines correspond to $\nu = 3$ and thin lines $\nu = 2$. Similar behavior can be observed for other gas evolution histories and model parameters.

By inspection of eqs. (1) and (2), the Comptonization parameter and the power spectrum scale linearly with the number density of clusters. Therefore, one expects that η will scale as $n_{cl}^{-1/2}$. In fig. 2b we plot $\eta n_{cl}^{1/2}$ for the same models as in Fig. 2a. Notice that the value of $\eta n_{cl}^{1/2}$ is almost constant and nearly independent on the threshold ν. The actual value varies by a factor of two when the total matter content changes from $\Omega_m = 1$ to 0.3 but does not depend on the geometry of the cosmological model, i.e. on the value of Λ. In Fig. 3 we plot η as a function of gas evolution history, assuming that the number density of all the objects contributing to the SZ effect is $2.5 \times 10^{-4} h^3 \mathrm{Mpc}^{-3}$. In each panel, thick lines correspond to a threshold $\nu = 3$ and thin lines to $\nu = 2$. Solid lines correspond to a lower mass integration limit of $10^{14} \mathrm{M}_\odot$ and dot-dashed lines to $10^{13} \mathrm{M}_\odot$. Notice that η varies by less than 30% for different gas evolutionary histories. Let as remark that, if $\Omega_m = \Omega_B + \Omega_{cdm}$ is known, η permits to obtain n_{cl}. Vice versa, if n_{cl} could be determined by other means, Ω_m could be determined. If no assumption is made about the cosmology, n_{cl} can be determined within a factor of four.

5. Discussion.

The results of previous section indicates that η is mostly sensitive to the number density of clusters, and for a given cosmological model is weakly dependent on the exact modeling of the cluster population. Therefore, a measurement of both the mean distortion of the CMB radiation and the temperature anisotropies induced by clusters of galaxies will permit to estimate the number density of objects that have enough hot gas to produce a detectable effect on the CMB. This conclusion is not limited by the validity of the scaling relations of Sec. [2]. One must expect a similar behavior for a different cluster model, even though the level of temperature anisotropies and distortions will be different. If the hot IC gas was uniformly distributed in the Universe, it would still produce blackbody distortions but will not give rise to temperature anisotropies, and η would be zero. Only when the gas is clumped in clusters, two different lines of sight, seeing different column densities, will have different temperatures. Therefore, $\bar{y}$ scales with the number of clusters along the line of sight while for Poisson distributed clusters $\Delta T/T_o$ scales as the square root and $\eta \propto n_{cl}^{-1/2}$, as shown. Therefore, independently of the exact model used to describe the average cluster properties, η measures how the hot gas is distributed in the Universe. Finally, if the cluster number density is known, η could be used as an estimator of the total matter density Ω_m.

To summarize, clusters of $10^{14} \mathrm{M}_\odot$ mass dominate the contribution to temperature anisotropies. Therefore, even if the adequacy of the scaling relations of Sec. [2] below that mass scale has not been established, temperature anisotropies can be reliably calculated using eqs. (2) and (3). If the entropy-driven model proves not to be adequate to describe groups of galaxies, one would have to use hydrodynamical simulations such as those of da Silva et al. (1999) to compute $\bar{y}$ but η would still provide an estimate of the number density of SZ clusters. Comparison of this density with that of X-ray clusters will undoubtably help our understanding of cluster formation and evolution.

REFERENCES

Atrio–Barandela, F. & Mücket, J. 1999, ApJ, 515, 465 (paper I)

Bahcall, N. A. 1999 in "Proceedings of the American Astronomical Society Meeting", 194, 3002

Barbosa, D., Bartlett, J. G., Blanchard, A., Oukbir, J. 1996, A& A, 314, 13

Bartlett, J.G., Silk, J. 1994, ApJ, 423, 12

Birkinshaw, M. 1999, Phys. Rep., 310, 97

Bower, R.G. 1997, MNRAS, 288, 355

Colafrancesco, S., Mazzotta, P., Rephaeli, Y. & Vittorio, N. 1994, ApJ, 433, 454

Cole, S. & Kaiser, N. 1989, MNRAS, 233, 637

Church, S. E. et al. 1997 ApJ, 484, 523

da Silva, A. C., Barbosa, D., Liddle, A. R. & Thomas, P. A. 1999, preprint astro-ph/9907224

Einasto, J. et al. 1999a, ApJ, 519, 441

Einasto, J. et al. 1999b, ApJ, 519, 456

Governato et al. 1999, MNRAS, submitted. Preprint astro-ph/9810189

Hobson, M. P., Jones, A. W., Lasenby, A. N., Bouchet, F. R. 1998, MNRAS, 300, 1

Jones, C. & Forman, W. 1984, ApJ, 276, 38

Komatsu, E. & Kitayama, T. 1999, preprint astro-ph/9908087

Markevitch, M., Forman, W.R., Sarazin, C.L. & Vikhlinin, A. 1997, ApJ, 503, 77

Mushotzky, R.F. & Scharf, C.A. 1997, ApJ, 482, L13

Perlmutter, S. et al. 1999, ApJ, 517, 565

Press, W.H., & Schechter, P. 1974, ApJ, 187, 425

Tozzi, P. & Governato, F. 1998, in "The Young Universe" S. D'Odorico, A. Fontana, and E. Giallongo, eds. ASP Conference Series, 146, 461

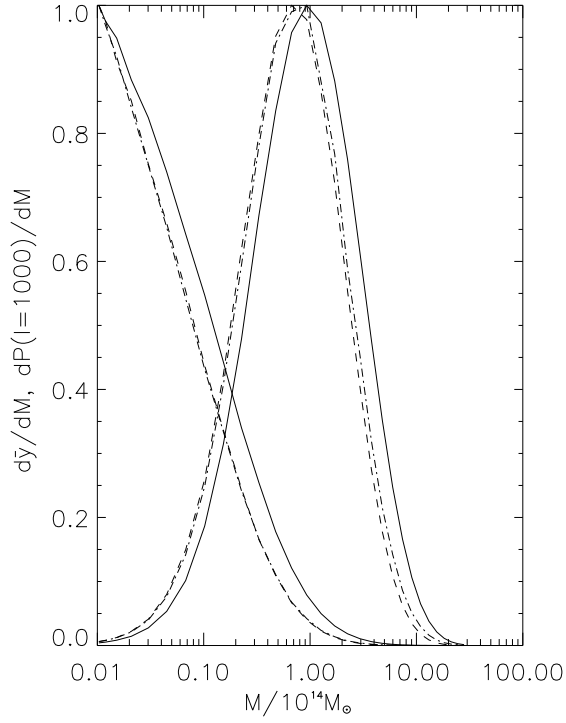Zel'dovich, Ya. B., Sunyaev, R. 1969, Ap&SS, 4, 301

Fig. 1.— Integrand of the mean Comptonization parameter (curves on the left side) and the radiation power spectrum (on the right) for different models. Solid lines correspond to standard CDM, dashed lines to ΛCDM and dot-dashed line to OCDM. The scale in the y-axis is arbitrary; the data has been rescaled for convenience. The amplitudes for $d\bar{y}/dM$ at $10^{12}M_\odot$ and dP/dM at the maximum are: 3×10^{-4}, 1.1×10^{-16} for sCDM, 1.5×10^{-4}, 1.4×10^{-17} for OCDM and 1.3×10^{-4}, 1×10^{-17} for ΛCDM. All these quantities are in units of $(M/10^{15}M_\odot)^{-1}$ and we took $g(x) = 1$.
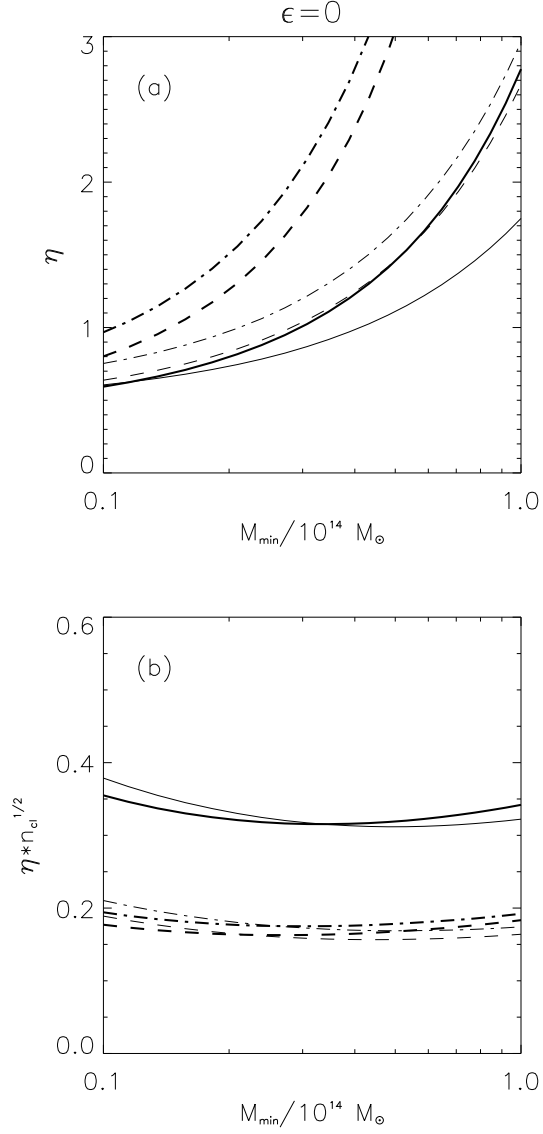
$$\epsilon = 0$$

Fig. 2.— (a) Variation of η (three upper curves) with the lower mass limit of integration M_{min} for three different cosmological models: SCDM (solid line), OCDM (dot-dashed line) and ΛCDM (dashed line). Thick lines correspond to a threshold $\nu = 3$ and thin lines to $\nu = 2$. (b) Variation of $\eta n_{cl}^{1/2}$ a function of M_{min} for the same cosmological models as in (a).
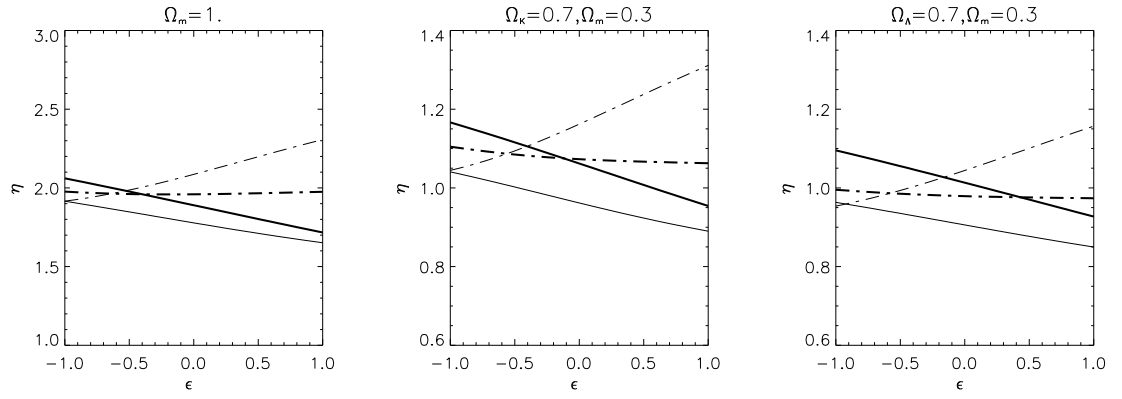
Fig. 3.— The value of η rescaled to a cluster density of $2.5 \times 10^{-4} h^3 \mathrm{Mpc}^{-3}$ clusters, as a function of the gas evolutionary history. From left to right: sCDM, OCDM and ΛCDM. Thick lines correspond to $\nu = 3$ and thin lines to $\nu = 2$; solid lines to $M_{min} = 1$ and dot-dashed lines to $M_{min} = 0.1$ (in units of $10^{14} \mathrm{M}\odot$).